The Primitives of Vector Calculus

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Area 1 - Points, Planes, and Vectors		
Level I	Level II	Level III
Area 1.1.I: Points in 3D Space	**Area 1.1.II**: Patterns of Points	**Area 1.1.III**: Relations of Points
Area 1.2.I: Planes in 3D Space	**Area 1.2.II**: Double-Slope of a Plane	**Area 1.2.III**: Planes Arising from Data Sets
Area 1.3.I: Vectors Based At the Origin	**Area 1.3.II**: Vectors Not Based at the Origin	**Area 1.3.III**: Vector Fields
Area 1.4.I: Addition and Multiplications of Vectors	**Area 1.4.II**: Addition and Multiplications of Non-Origin-Based Vectors	**Area 1.4.III**: The 4D Vectors: Quaternions
Area 1.5.I: Basic Relationships Between Vectors and Planes	**Area 1.5.II**: The Normal Vector to a Plane	**Area 1.5.III**: Normal Vector Field to a Plane

Area 2 - Visual Investigation of Curves and Surfaces		
Level I	Level II	Level III
Area 2.1.I: 3D Surfaces: Geometric, Algebraic, Numerical	**Area 2.1.II**: Catalog of 3D Surfaces	**Area 2.1.III**: 4D Hypersurfaces and Their 3D Slices and Projections
Area 2.2.I: Slicing 3D Surfaces	**Area 2.2.II**: Families of Intersection Curves	**Area 2.2.III**: Intersections Between Two 3D Surfaces
Area 2.3.I: 2D Contour Diagrams of 3D Surfaces	**Area 2.3.II**: Numerical Descriptions of 3D Functions and Contours	**Area 2.3.III**: Contours of 4D Hypersurfaces
Area 2.4.I: Optimization of $F(x, y)$ via 3D Graphs and Contours	**Area 2.4.II**: Optimization via Numerical Contour Tables	**Area 2.4.III**: Applications Utilizing Graphical Optimization
Area 2.5.I: Polar, Cylindrical, and Spherical Coordinates	**Area 2.5.II**: Catalog of Polar, Cylindrical, and Spherical Curves and Surfaces	**Area 2.5.III**: Parabolic, Hyperbolic, and Other Coordinate Systems
Area 2.6.I: 2D and 3D Parametric Curves	**Area 2.6.II**: Important Vectors Living on a Curve	**Area 2.6.III**: Vector Fields on a Curve
Area 2.7.I: Parametric Surfaces	**Area 2.7.II**: Catalog of Parametric 3D Surfaces	**Area 2.7.III**: Intersection Curves of Parametric Surfaces

Area 3 - Differentiation of 3D Surfaces		
Level I	Level II	Level III
Area 3.1.I: Partial Derivatives of Functions $F(x, y)$	**Area 3.1.II**: Partial Derivatives of Numerically-Defined Functions	**Area 3.1.III**: Partial Derivatives as 3D Surfaces
Area 3.2.I: Geometric Directional Derivatives	**Area 3.2.II**: Numerical Directional Derivatives	**Area 3.2.III**: Optimization of Directional Derivatives
Area 3.3.I: THE Derivative of $F(x, y)$	**Area 3.3.II**: The Numerical and Algebraic First Derivative Test	**Area 3.3.III**: Translated Gradient Normal Field to a 3D Surface
Area 3.4.I: THE Second Derivative of $F(x, y)$	**Area 3.4.II**: The Numerical and Algebraic Second Derivative Test	**Area 3.4.III**: Unclassifiable Points on a 3D Surface
Area 3.5.I: Geometric Optimization of Surface Intersection Curves	**Area 3.5.II**: Algebriac Optimization of Surface Intersection Curves	**Area 3.5.III**: Generalizations and Second Derivative Tests for Lagrange Multipliers
Area 3.6.I: Linear Appoximation to a 3D Surface at a Point	**Area 3.6.II**: Linearizing Data Tables	**Area 3.6.III**: Higher-Order Polynomial Approximations to 3D Surfaces

Area 4 - Integration of 3D Surfaces		
Level I	Level II	Level III
Area 4.1.I: Riemann Sums of Functions $F(x, y)$ Over Rectangular Regions	**Area 4.1.II**: Trapezoidal and Simpson's Sums for Functions $F(x, y)$	**Area 4.1.III**: Iterative Numerical Methods to Calculate Definite Double Integrals
Area 4.2.I: Riemann Sums for Double Integrals over Non-Rectangular Regions	**Area 4.2.II**: Numerical Double Integrals Over Challenging Regions	**Area 4.2.III**: Error in Numerical Integral Calculations
Area 4.3.I: Algebraic Antiderivatives Applied to Definite Integrals	**Area 4.3.II**: Reversing the Order of Algebraic Integration	**Area 4.3.III**: Functions Whose Antiderivatives are Not Algebraic
Area 4.4.I: u-v-substitution for Double Integrals	**Area 4.4.II**: Volume Elements in General Coordinate Transformations	**Area 4.4.III**: Generalizations of Integration by Parts to Double Integrals
Area 4.5.I: Integration Over Polar, Cylindrical, and Spherical Coordinates	**Area 4.5.II**: Completing Algebraic Integration via Coordinate Conversion	**Area 4.5.III**: Volume Elements in Other Coordinate Systems

Area 5 - Analysis of 2D Vector Fields		
Level I	Level II	Level III
Area 5.1.I: Two Things Vector Fields Like to Do	**Area 5.1.II**: Linear Vector Fields	**Area 5.1.III**: Local Linearity of Any Vector Field
Area 5.2.I: Measurements of Sinks, Sources, and Curly Points	**Area 5.2.II**: Classification of Sinks, Sources, and Curly Points	**Area 5.2.III**: Viewing Curl and Div as 3D Surfaces
Area 5.3.I: Exterior Differentiation of a Vector Field	**Area 5.3.II**: Formal Development of Differential Forms	**Area 5.3.III**: Exterior Algebras
Area 5.4.I: Integration of 2D Vector Fields over Parametric Curves	**Area 5.4.II**: Interpretation as Measurement of Flow	**Area 5.4.III**: Measurement of Flow With Singularities
Area 5.5.I: Antiderivatives of Vector Fields and FTOC	**Area 5.5.II**: Conservative Vector Fields and FTOC	**Area 5.5.III**: Singularities and Their Affect on FTOC

Area 6 - Analysis of 3D Vector Fields		
Level I	Level II	Level III
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Area 6.2.I: Exterior Derivatives in 3D: Measurement of Sinks, Sources, Curlys	**Area 6.2.II**: Antiderivatives of 3D Vector Fields	**Area 6.2.III**: Classification of Sinks, Sources, and Curly Vectors in 3D
Area 6.3.I: Integration of Vector Fields Over Parametric Curves in 3D	**Area 6.3.II**: Integration over Conservative Vector Fields	**Area 6.3.III**: Affects of Singularities on Integration in 3D
Area 6.4.I: Integration of 3D Vector Fields over Parametric Surfaces	**Area 6.4.II**: Integration over Conservative Vector Fields	**Area 6.4.III**: Affects of Singularities on 3D Surface Integrals
Area 6.5.I: FTOC for 3D Vector Fields	**Area 6.5.II**: Higher Dimensional FTOC	**Area 6.5.III**: Singularities, p-simplicies and the FTOC
Area 6.6.I: Practical Use of 3D FTOC	**Area 6.6.II**: Physics Likes to Work With Boundary	**Area 6.6.III**: